サンプル

令和X年度　Y月実施
筑波大学大学院　入学試験
理工情報生命学術院　数理物質科学研究群
応用理工学学位プログラム
電子･物理工学サブプログラム　試験問題

専門科目

注意事項（選択、解答についての必要な指示）

1. 問題は 6 題あり、このうち問題 1 は必ず解答しなさい。また問題 2 から問題 6 までの中から 3 題を選び解答しなさい。ただし、問題 2 または問題 3 のどちらかは必ず解答すること。
2. 日本語または英語で解答すること。
3. 問題 1 は問(1)、問(2)から成り、それぞれ別の問題用紙に記載されています
4. 4 題を超えて解答したときは、すべての答案を無効とします。
5. **答案用紙全てに受験番号を記入すること。**
6. 答案用紙は全部で 5 枚あります。

   問題 1 の問(1)、問(2)は、答案用紙の最初の 2 枚に問題番号が指定されているので、それぞれ該当する用紙に解答しなさい。

   問題 2 から問題 6 の選択問題は、残り 3 枚の答案用紙を使って 1 題を 1 枚の答案用紙に解答しなさい。この場合、**各答案用紙の左上に必ず問題番号を記入**しなさい。紙面が足りないときは裏面を使うこと。
7. 答案用紙のホチキス針は取り外さないこと。

20XX Entrance Examination of
Degree Programs in Pure and Applied Sciences
Master's Program in Engineering Sciences:
Subprogram in Applied Physics

Specialized Subjects

Notes:

1. There are six problems given. Problem 1 is COMPULSORY. From Problems 2 to 6, select THREE problems and solve them. Either or both Problems 2 and 3 have to be selected.
2. Problem 1 consists of two parts, (1) and (2), each given on a separate sheet.
3. **Answers must be in Japanese or English.**
4. If you give answers to more than four problems, the exam will be invalid.
5. **Write the examinee's number on every answer sheet.**
6. You have FIVE answer sheets. For Problem 1, use the first two sheets on which the respective question number have been given. The other three sheets are for the three problems you select among Problem 2 to 6. You must use a single sheet for each of the problems and indicate the problem number at the top left corner of the sheet. If space on the answer sheet is not enough, use the back of the sheet.
7. Do not remove the staples of the answer sheets.

Applied Physics

Problem 1 (140 points) Mathematics [数学] 1/2

Note: Problem 1 has two parts, (1) and (2). Answer all of them.

(1) Answer the following questions (a) and (b). Let x, y, t, θ be real numbers [実数], z be a complex number [複素数], and i be the imaginary unit [虚数単位].

(a) Answer the following questions.

(i) Find the double integral [2 重積分]

$$\iint_D (2x+y)^3 dxdy, \quad D = \{(x, y) | 0 \le 2x + y \le 1, \ 0 \le x - y \le 1\}.$$

(ii) Evaluate $\lim_{x \to 0} \dfrac{x - \sin x}{(1-e^x)(1-\cos x)}$.

(iii) Evaluate $\lim_{x \to +0} (\sin x)^x$.

(b) Answer the following questions related to the definite integral [定積分]

$$I = \int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta}.$$

(i) Show that $\cos\theta = \dfrac{1-t^2}{1+t^2}$, $d\theta = \dfrac{2dt}{1+t^2}$, when $\tan\dfrac{\theta}{2} = t$.

(ii) Find the definite integral I by using (i).

(using $\displaystyle\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = 2\int_0^{\pi} \frac{d\theta}{5+3\cos\theta}$ is allowed)

(iii) Express the definite integral I in terms of the complex variable $z = e^{i\theta}$ by transforming it into an integral along the unit circle C in the complex plane [複素平面上の単位円Cを経路とする積分に変換せよ].

(iv) Find the definite integral I in the complex plane [複素平面] by using (iii).

(continued to part (2) on the next page)

Applied Physics

Problem 1 (140 points) Mathematics [数学] 2/2

Note: Problem 1 has two parts, (1) and (2). Answer both parts.

(2) Answer the following questions concerning a 3×3 matrix $A = \begin{pmatrix} a & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}$.

(a) Calculate $\mathrm{tr}\, A$.

(b) Calculate $\det A$.

In the following, consider the case where $a = 3$ and $b = -1$.

(c) Find the eigenvalues [固有値] λ_1, λ_2 and λ_3 of A where $\lambda_1 \leq \lambda_2 \leq \lambda_3$.

(d) Find the normalized eigenvectors [正規化された固有ベクトル] $\boldsymbol{u}_1, \boldsymbol{u}_2$, and $\boldsymbol{u}_3$ of A where $A\boldsymbol{u}_i = \lambda_i \boldsymbol{u}_i$ $(i = 1, 2, 3)$.

(e) Find the diagonal matrix D[対角行列], the orthogonal matrix P[直交行列], and P^{-1} by which A is diagonalized as $P^{-1}AP = D$.

(f) Calculate D^n where n is a positive integer [整数].

(g) Calculate A^n where n is a positive integer.

(h) Evaluate $\mathrm{tr}(\cos A)$ where $\cos A$ is defined as follows.

$$\cos A = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} A^{2n}$$

Applied Physics

Problem 2 (120 points) Mechanics [力学]

A cylinder［円柱］ with mass M, radius a, length L, and constant density ρ is shown in Fig.2-1. The origin [原点] O is the center of gravity [重心] of the cylinder; the z - axis is taken along the central axis; and the x and y - axes are taken in directions perpendicular to the cylinder. In addition, the position of an arbitrary point [任意の点] P(x, y, z) inside the cylinder is also represented by cylindrical coordinates [円筒座標] (r,θ,z). Here, $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$. I_z and I_x are the moment of inertia [慣性モーメント] of the cylinder along the z and x - axis, respectively.

Answer the following questions.

(1) Find the expression of I_z using the infinitesimal volume［微小部分の体積］ $r \cdot dr \cdot d\theta \cdot dz$ of the cylinder.

(2) Show that $I_z = \frac{Ma^2}{2}$.

(3) As shown in Fig. 2-2, a constant magnitude of force F_1 is applied to point P_1［点 P_1 に一定の大きさの力F_1を作用させる］ on the surface of the cylinder (assuming that point P_1 is in the xy plane), and the cylinder rotates along the z - axis. The direction of the force F_1 is always perpendicular to both the axis of rotation and $\overrightarrow{\mathrm{OP_1}}$. Once the force has started to act, and after time t, find the expression of the rotation angle θ_z in terms of F_1, M, a, and t.

(4) In the case of (3), find the time t_1 for the cylinder to rotate once.

(5) Show that $I_x = \frac{Ma^2}{4} + \frac{ML^2}{12}$.

(6) Use a to express L when $I_x = I_z$.

(7) In the case of (6), as shown in Fig. 2-3, a constant magnitude of force F_2 is applied to point P_2 on the surface of the cylinder (assuming that point P_2 is in the yz plane and the distance between the origin O and the point P_2 is ℓ) to rotate the cylinder about the x - axis. The direction of force F_2 is always perpendicular to both the axis of rotation and $\overrightarrow{\mathrm{OP_2}}$. In this case, to rotate the cylinder exactly once at the same time as t_1 in (4), it is necessary to satisfy a certain condition of force F_2. Use F_1 to express the minimum force F_2 that satisfies this condition. In that case, use a to express the distance ℓ.

(continued on the next page)

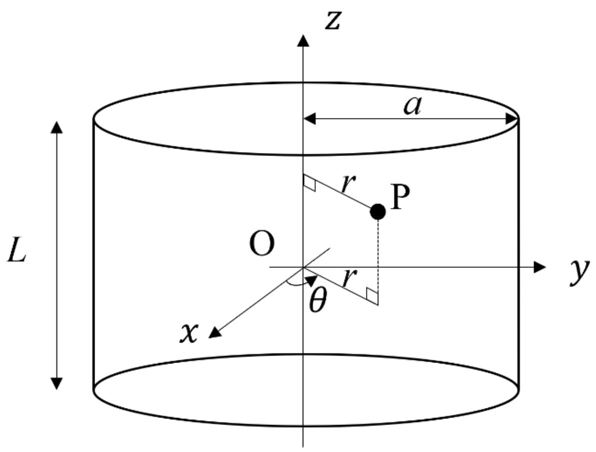


Fig.2-1

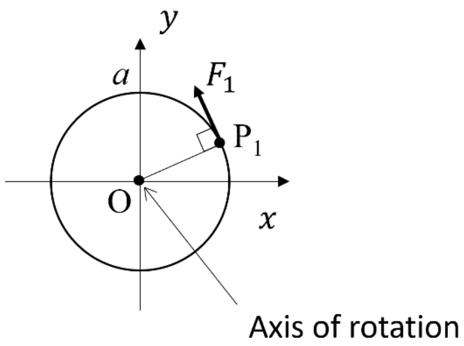


Fig.2-2

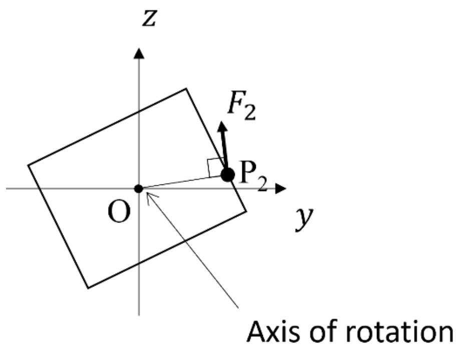


Fig.2-3

Applied Physics

Problem 3 (120 points) Electromagnetism [電磁気学]

As shown in Fig.3-1, suppose that there are two long coaxial conductive hollow cylindrical tubes [中空の導体円管] A and B with radii a and b $(a < b)$ in the vacuum space (permittivity: ε_0, permeability: μ_0). Here, the lengths of both cylindrical tubes are much larger than a or b, and the thicknesses and the effects of their edge of the cylindrical tubes may be ignored [円管の厚さおよび端の影響は無視できる]. Let r be the radial distance from the central axis of the cylindrical tubes [円管の中心軸からの距離]. Answer the following questions.

First, as shown in Fig. 3-2, consider the case where the inner cylindrical tube A is charged with $+\lambda$ per unit length [単位長さあたり], and the outer cylindrical tube B is grounded [接地].

(1) Find the electric field strength in the region inside the cylindrical tube A $(0 \leq r < a)$.

(2) Find the electric field strength in the region between the cylindrical tubes $(a < r < b)$.

(3) Find the potential difference between the two cylindrical tubes.

(4) Find the capacitance per unit length of this coaxial structure [同軸構造の単位長さあたりの静電容量].

(5) Find the electrostatic energy stored per unit length of this coaxial structure [同軸構造の単位長さあたりに蓄積される静電エネルギー].

Next, as shown in Fig. 3-3, consider that at one end both the cylindrical tubes are connected together by a resistor [抵抗] and at the other end they are connected to a power source [電源]. Also consider that a constant current I is flowing through both cylindrical tubes in opposite directions to each other. Here, the current density of each cylindrical tube is uniform [一様].

(6) Find the magnetic flux density [磁束密度] in the region between the two cylindrical tubes $(a < r < b)$.

(7) Find the self-inductance per unit length [単位長さあたりの自己インダクタンス] of this coaxial structure.

(8) Find the magnetic energy stored per unit length of this coaxial structure.

(continued on the next page)

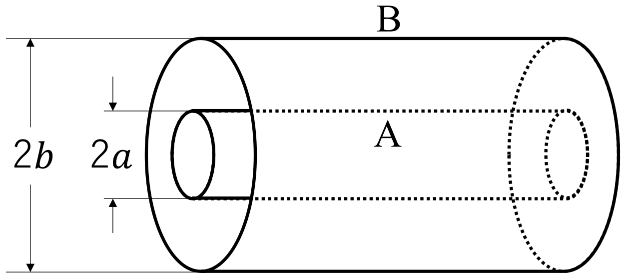


Fig. 3-1

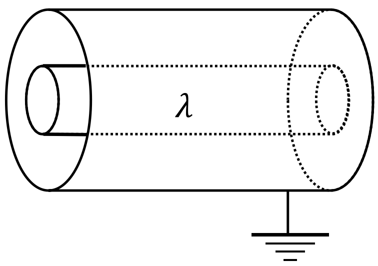


Fig. 3-2

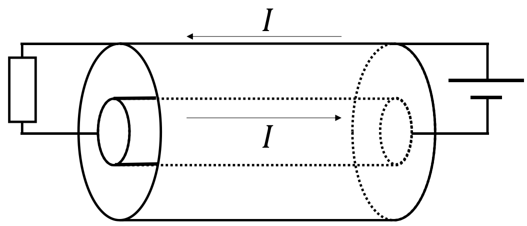


Fig. 3-3

Applied Physics

Problem 4 (120 points) Quantum Mechanics [量子力学]

Consider the motion of a particle with mass m confined to a one-dimensional infinite square well potential [1 次元の無限井戸型ポテンシャル] (well width a). The potential $V(x)$ is given as follows, where $a > 0$:

$$V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & x < 0, a < x \end{cases}$$

The wave function $\psi(x)$ of the particle in the ground state [基底状態] of this potential $V(x)$ is expressed as follows in the range $0 \leq x \leq a$: $\psi(x) = A \sin\left(\frac{\pi x}{a}\right)$. Here, A is a constant. Answer the following questions. In (2) and later, you may use A as it is [ただし、(2)以下では A をそのまま使って良い].

(1) Find the normalization constant [規格化定数] A for the wave function $\psi(x)$ of the particle.
(2) Determine the energy E of the particle.
(3) Write the mathematical expression for calculating the expectation value $\langle x \rangle$ of the position of the particle [粒子の位置の期待値$\langle x \rangle$]. Also, find the value of $\langle x \rangle$.
(4) Calculate the uncertainty Δx in the position of the particle [粒子の位置の不確かさΔx]. You may use the fact that
$\langle x^2 \rangle = \frac{a^2}{6}(2 - \frac{3}{\pi^2})$.
(5) Express mathematically the calculation of the expectation value, $\langle p \rangle$, of the momentum of the particle [粒子の運動量の期待値$\langle p \rangle$]. Also, find the value of $\langle p \rangle$. You may use the fact that $\int_0^a \sin\frac{\pi x}{a} \cos\frac{\pi x}{a} dx = 0$.
(6) Determine the uncertainty Δp in the momentum of the particle [粒子の運動量の不確かさΔp].

Applied Physics

Problem 5 (120 points) Optics [光学]

(1) Here we consider a complex electric field [複素光電場] $f_1(t,z) = a\exp[i(\omega t - kz)]$, where a is a real positive constant, t is a variable representing time, z is a variable representing one-dimensional space position, and ω and k are the angular frequency [角振動数] and the wavenumber [波数] of this electric field, respectively.

(a) Write the wavelength of this electric field.

(b) The propagation direction and the velocity of light are defined as the moving direction and velocity of the equi-phase plane [等位相面] of the corresponding electric field. Here the equi-phase plane is a plane in which the phase of the field is constant. Verbally describe (i.e., describe by sentences) how you can obtain (i.e., compute) the propagation direction and the speed of the complex electric field defined above.

(c) Show (i.e., compute) the propagation direction of this electric field.

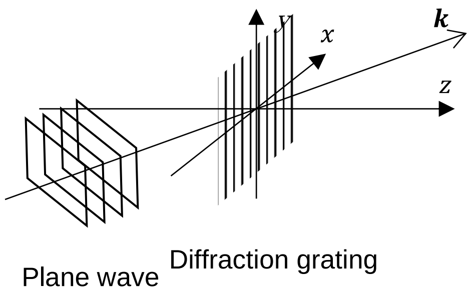
(d) Show (i.e., compute) the propagation velocity of this electric field.

(2) Now we consider a complex electric field propagating in a three-dimensional space.

(e) We express the three-dimensional position by a vector $\boldsymbol{r} = (x,y,z)$ and the propagation direction by a vector $\boldsymbol{k} = (k_x, k_y, k_z)$. Write the equation representing an electric field of monochromatic plane wave [単色平面波] $f_2(t,\boldsymbol{r})$ which propagates to the direction of $\boldsymbol{k}$ and has the amplitude of a and an angular frequency ω. Note that we assume that the initial phase (i.e., the phase at $t = 0$ and $\boldsymbol{r} = (0,0,0)$) is 0.

(f) Now we consider the monochromatic plane wave obtained in (e), whose propagation direction is represented with $\boldsymbol{k}$, is incident on a diffraction grating from the left as show in the figure below. Namely, the wave is incident on the grating from the z < 0 side, and $k_z > 0$. The diffraction grating [回折格子] is at the plane of $z = 0$, and is thin enough to ignore its thickness. The transmittance [透過率] of the grating is described as $g(x) = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi x}{d}\right)\right]$. Write the wave (i.e., the electric field) just after the grating as an equation which uses $g(x)$ and $f_2(t,\boldsymbol{r})$.

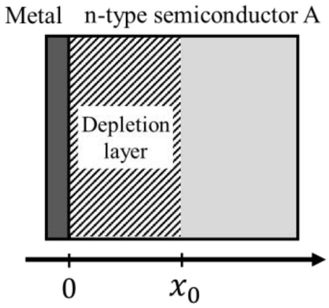
(g) Note that $1 + \cos\theta = 1 + \left(e^{i\theta} + e^{-i\theta}\right)/2$ in general. At first, verbally describe how you obtain the propagation directions of the transmitted light and the diffracted light from the grating of question. Second, compute and show the vectors representing these propagation directions.



Applied Physics

Problem 6 (120 points) Semiconductor Engineering [半導体工学]

Consider an n-type Schottky barrier diode (SBD-A) using a uniformly doped n-type semiconductor A. The donor concentration is N_D, and all donors are fully ionized. The position of the Schottky junction is defined as $x = 0$. The elementary charge [電荷素量] is q and the semiconductor permittivity is ε. Answer the following questions.



At thermal equilibrium [熱平衡状態], a depletion layer [空乏層] with width of x_0 and diffusion potential [拡散電位] V_D are formed in the n-type semiconductor A.

(1) Draw an energy band diagram of SBD-A. In the diagram, show the bottom of the conduction band E_C, the top of the valence band E_V, the Fermi level in the semiconductor E_Fs, the Fermi level in the metal E_Fm, and V_D.

(2) Express the Poisson's equation [ポアソン方程式] using the potential $\phi(x)$ in the depletion region $(0 \leq x \leq x_0)$.

(3) Assuming $\phi(0) = 0$, express the electric field $E(x)$ and the potential $\phi(x)$ in the depletion region.

(4) Demonstrate $x_0 = \sqrt{\frac{2\varepsilon V_\mathrm{D}}{qN_\mathrm{D}}}$.

A reverse bias $V(< 0)$ is now applied to SBD-A. When $V = V_1$, the breakdown [絶縁破壊] of SBD-A occurs. In this condition, the maximum electric field and the breakdown voltage are E_1 and $|V_1|$, respectively. Here, the diffusion potential V_D is neglected.

(5) Express the maximum depletion layer width x_1 and E_1.

Consider an n-type Schottky barrier diode (SBD-B) using an n-type semiconductor B. The breakdown field of the semiconductor B is 10 times higher than that of the semiconductor A. Let the breakdown voltage of SBD-B be the same as that of SBD-A. Here, the permittivity and the electron mobility of the semiconductor B are assumed to be the same as those of the semiconductor A.

(6) Express the donor concentration of the semiconductor B in terms of N_D.

(7) Describe how much the depletion layer width in the semiconductor B is reduced. Considering the change in the donor concentration and the depletion layer width, discuss how much the resistance of the blocking layer under forward bias conditions is reduced comparing to the semiconductor A.